On the Use of Yielded Cost in Modeling Electronic Assembly Processes

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Abstract – Yielded cost is defined as cost divided by yield and can be used as a metric for representing an effective cost per good (non-defective) assembly for a manufacturing process. Although yielded cost is not a new concept, it has no consistent definition in engineering literature, and several different formulations and interpretations exist in the context of manufacturing and assembly.

In manufacturing, yield is the probability that an assembly is non-defective. To find the effective cost per good assembly that is invested in the manufacturing or assembly process, cost is accumulated and divided by the yield at the end of the process.

This paper reviews and correlates existing yielded cost formulations and presents a new approach that enables consistent measurement of sequential process flows. This new approach defines the yielded cost associated with an individual process step (step yielded cost) as the change in the process’s yielded cost when the step is removed from the process. This approach is preferred because it incorporates upstream and downstream information and because it provides a prediction of a specific process step’s effective cost per good assembly that is independent of step order between steps that scrap defective product.

Index Terms – cost, yield, yielded cost, design to cost, rework, test economics.

NOMENCLATURE

C_{dr} cost incurred per assembly in the diagnosis/rework step.
C_{i} cost incurred per assembly in the \(i^{th}\) step in a sequential process flow.
C_{in} cost per assembly prior to a process flow.
C_{int} total cost invested by the test and diagnosis/rework steps in a rework process over \(n_{t}\) rework attempts.
C_{out} accumulated cost per assembly following a process flow.
C_{step} cost incurred per assembly in a general process step.
C_{test} cost incurred per assembly in a test step.
C_{j}\_tj accumulated cost per assembly following the \(j^{th}\) test step.
C_{Y} yielded cost.
C_{Yi} yielded cost of the \(i^{th}\) step in a sequential process flow.
C_{Y_{(i-1)} to i} yielded cost prior to step \(i\).
C_{Y_{i to (i+1)}} yielded cost following step \(i\).
C_{Yin} yielded cost prior to a process flow.
C_{Y_{int}} yielded cost of a rework process with \(n_{t}\) rework attempts.
C_{Y_{out}} yielded cost following a process flow.
C_{Step} process step yielded cost.
C_{Y_{j}\_tj} yielded cost following the \(j^{th}\) test step.
C_{Y_{test}} yielded cost of a test step.
C_{Y_{total}} process yielded cost.
\(f_{c}\) fault coverage (fraction of faults present in an assembly that are detected by a test step).
\(f_{dr}\) fraction of assemblies reworked by the diagnosis/rework step.
i subscript denoting the \(i^{th}\) step in a process flow.
m number of steps in a process flow.
n subscript denoting the \(n^{th}\) rework iteration.
n_{t} the maximum number of rework attempts on a single assembly.
N_{drn} number of assemblies sent from diagnosis/rework to test on the \(n^{th}\) rework attempt.
**N**<sub>g</sub> total number of good assemblies passing a test step after n<sub>r</sub> rework attempts.

**N**<sub>outn</sub> number of assemblies passing the test step on the n<sup>th</sup> rework attempt.

**N**<sub>sdrn</sub> number of assemblies scrapped by diagnosis/rework on the n<sup>th</sup> rework attempt.

**N**<sub>n</sub> number of assemblies sent from test to diagnosis/rework on the n<sup>th</sup> rework attempt.

**P** pass fraction (fraction of parts that are passed by a test step).

**Y**<sub>a</sub> yield prior to a test step.

**Y**<sub>b</sub> yield following a test step.

**Y**<sub>dr</sub> yield of a diagnosis/rework step (due to defects introduced by the diagnosis/rework operation).

**Y**<sub>i</sub> yield of the i<sup>th</sup> step in a sequential process flow.

**Y**<sub>in</sub> yield of assemblies prior to a process flow.

**Y**<sub>out</sub> yield of assemblies following a process flow.

**Y**<sub>step</sub> yield of a process step.

**Y**<sub>test</sub> yield of a test step (due to defects introduced by the test operation).

**Y**<sub>ij</sub> yield following the j<sup>th</sup> test step.

(1 - **Y**<sub>i</sub>) auxiliary cost factor due to i<sup>th</sup> step yield.

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**I. INTRODUCTION**

To date, industry has left yielded cost (cost divided by yield) formally undefined and has not fully embraced its meaning, usefulness and ramifications. For many years, however, engineers have used yielded cost in manufacturing cost analyses as a method of measuring the effective cost of processes. It has been referred to under several different names, such as yielded die cost [1] or chip set cost [2], [3] and its application has depended upon the specific manufacturing process under analysis. As a result, much of its value as a general diagnostic and quality evaluation metric is lost. If defined properly, however, yielded cost could be used to consistently and accurately to determine the effective contribution of individual process steps to entire processes, and could thus more effectively identify critical steps. Manufacturers could then improve process quality and performance-price ratios [4] and use yielded cost to improve manufacturing and assembly processes.

Yielded cost, in general, is described as *cost divided by yield*, Fig. 1. One can appreciate the value of this definition by considering an example: if C<sub>m</sub> = 0, Y<sub>m</sub> = 1.0, setting C<sub>i</sub> = 100 and Y<sub>i</sub> = 0.9 for m = 3 steps in Fig. 1, gives C<sub>Y</sub> = $300/(0.93) = $412 per good assembly. This measurement is valuable because it represents an effective cost per good assembly after three process steps, which helps in evaluating the overall quality of the process.

A close look at the electronic and mechanical systems cost modeling literature indicates that cost divided by yield appears frequently, examples include integral passive modeling [5], yield prediction and associated cost for printed circuit packs [6], integrated optical chips [7], VLSI floorplanning [8], flip chip and wire bonding [9], expected profit models for multi-stage manufacturing systems [10], the implementation of inspection costs for optimal lot sizing [11], and cost of ownership (COO) applications [12] and [13]. Actual references to the specific concept of yielded cost have also appeared in the literature, mostly as a means of developing cost models, e.g., Matsuno et al. [1] addresses yielded cost in a paper on the development of a yield and cost-forecasting model for monolithic microwave integrated circuits (MMICs). Although none of these references define the concept incorrectly, previous work as a whole has inconsistently applied yielded cost, and has therefore limited the potential usefulness of the concept. Also, the usefulness of yielded cost has also been stifled because no attempt has been made to correlate individual step yielded costs to overall process yielded costs, and yielded cost has never been extended to test and rework applications.

In order to address these issues, this paper will evaluate existing definitions and derive a more appropriate yielded cost metric. Section II of this paper guides the reader through process flow examples to demonstrate the meaning of yielded cost and compare alternative definitions. Section III addresses test issues and Section IV explains how step yielded cost components are distributed in a process. Section V treats rework and Section VI applies the yielded cost approaches to a surface mount process flow.
II. DEFINITION AND USE OF YIELDED COST

In process-flow analysis, manufacturing operations are typically analyzed as a series of fabrication and assembly steps, each with specific costs and yields. The step costs typically account for material, labor, tooling, equipment, and facilities [14] while the yields are determined through sampling [15] with some tolerance [16]. Process yield is defined as the number of usable assemblies after manufacturing divided by the number of assemblies that start the manufacturing process.

One way to characterize the quality of a process is with yielded cost. Process yielded cost, $C_{Y_{total}}$, introduced in Fig. 1, characterizes the quality of the entire process under consideration and is defined as the total cost invested per assembly divided by the yield at the end of the process. Step yielded cost, $C_{Y_{step}}$, derives from $C_{Y_{total}}$ and represents the effective cost contribution of an individual process step towards the process as a whole. Although process yielded cost has been used consistently in the past, step yielded cost has not. Therefore, an appropriate method of computing step yielded cost must be found. The criteria used for evaluating an approach are: 1) one must be able to collect step yielded costs in some way to obtain process yielded cost, 2) step yielded costs must account for both upstream and downstream process information, and 3) step yielded costs must be independent of step order between “scrapping steps,” where assemblies are removed from the process (i.e., test or inspection steps).

Collection of step yielded costs is necessary because the sum of effective cost contributions should represent the effective cost of the entire process itself. Incorporating upstream and downstream information is necessary because step yielded cost should account for both a step’s effect on all other process steps and all other process steps’ effect on the step under consideration. Lastly, independence of step order, for steps between scrapping points, is necessary because cost can be incurred on defective assemblies or on assemblies to be made defective. This is further explained in Part B of this section. Four approaches to calculating step yielded cost have been identified: the itemized, iterative, cumulative, and omission methods. The first criterion was met by the cumulative and omission methods while it was not met with the itemized and iterative methods. Additionally, the omission method was the only approach found to satisfy the second and third criteria.

The itemized approach, simply defines $C_{Y_{step}}$ as $C_{step}$ divided by $Y_{step}$. In Fig. 1, with this definition, some $C_{Y_{step}}$ values are $C_{Y_{in}} = C_{in}/Y_{in}$ and $C_{Y_1} = C_1/Y_1$. The yielded cost following step 1 would then be $C_{Y_{in}} + C_1/Y_1$. Since this is not equal to $C_{Y_{total}} = (C_{in} + C_1)/Y_{in}Y_1$, this approach does not satisfy the first criteria ($C_{Y_{step}}$ values cannot be collected to get $C_{Y_{total}}$). Furthermore, with the iterative approach used by Matsuno et al. [1], the yielded cost following some step $i$, $C_{Y_{i to (i+1)}}$, is the yielded cost prior to step $i$, $C_{Y_{(i-1) to i}}$, plus the cost incurred per assembly in step $i$, $C_i$, all divided by the step yield, $Y_i$.

$$C_{Y_{i to (i+1)}} = \frac{C_{Y_{(i-1) to i}} + C_i}{Y_i} \quad (1)$$

Then $C_{Y_{step}}$ for step $i$ is defined as $C_{Y_{i to (i+1)}} - C_{Y_{(i-1) to i}}$. This approach also does not satisfy the first criteria because $C_{Y_{step}}$ values cannot be collected to get $C_{Y_{total}}$.

A. Cumulative Approach to Yielded Cost

Similar to the iterative approach, the cumulative approach [17] defines $C_{Y_{step}}$ as the yielded cost following the step minus the yielded cost prior to the step; however, yielded cost is defined as in Fig. 1, not by (1). Using the cumulative approach, the $C_{Y_{step}}$ values in Fig. 2 are given by,

$$C_{Y_{in}} = \frac{C_{in}}{Y_{in}} \quad (2)$$

$$C_{Y_1} = C_{Y_{in}} - C_{Y_{in}} = \frac{C_{in} (1 - Y_1) + C_1}{Y_{in} Y_1} \quad (3)$$

$$C_{Y_2} = C_{Y_{in}} - C_{Y_{in}} = \frac{(C_{in} + C_1) (1 - Y_2) + C_2}{Y_{in} Y_1 Y_2} \quad (4)$$

With the assumption that no processing occurs before step 1, the total cost and yield before step 1 would be equal to $C_{in}$ and $Y_{in}$ respectively. Thus, (2) also represents the yielded cost entering the process. This
approach is reasonable because the $C_{Y_{\text{step}}}$ values expressed in (2), (3), and (4), can be summed to get $C_{Y_{\text{out}}}$ shown in Fig. 2. However, the $C_{Y_{\text{step}}}$ values are blind to downstream information (i.e., the effects of processing that takes place after the current step) by the nature of this calculation. For example, the expression for $C_{Y_1}$ in (3) does not consider the effects of step 2 (no $C_2$ or $Y_2$ terms). With a decrease in $Y_2$, for example, a greater proportion of the cost invested in step 1 would be spent on the assemblies made defective in step 2. So, $C_{Y_1}$ should incorporate these effects. Additionally, the cumulative method's $C_{Y_{\text{step}}}$ values are not independent of step order. If step 1 and step 2 of Fig. 2 were switched, for example, then $C_{Y_1}$ would change to resemble (4).

Because the cumulative method does not consider downstream information and its values are not independent of step order, it falls short of completely describing step yielded cost.

**B. Omission Approach to Yielded Cost**

A new method, the omission method defines $C_{Y_{\text{step}}}$ as the difference between $C_{Y_{\text{total}}}$ computed with the step in the process flow and $C_{Y_{\text{total}}}$ computed without the step in the process flow. The step yielded costs calculated with this method represent the change in process yielded cost obtained by removing a step from the process flow. Under this definition, the yielded cost of the first step in Fig. 2 would be,

$$C_{Y_1} = \frac{C_{\text{in}} + C_1 + C_2}{Y_{\text{in}} Y_1 Y_2} - \frac{C_{\text{in}} + C_2}{Y_{\text{in}} Y_2} = \frac{C_{\text{in}} (1 - Y_1) + C_1 + C_2 (1 - Y_1)}{Y_{\text{in}} Y_1 Y_2} \tag{5}$$

Similar to the cumulative method, $C_{Y_{\text{step}}}$ values obtained with omission can be collected to get $C_{Y_{\text{total}}}$. If the numerator of (5) is separated, the second term, $C_{\text{in}} Y_{\text{in}} Y_1 Y_2$, is the base cost (the effective cost invested in the step of interest). The sum of the base costs for each respective $C_{Y_{\text{step}}}$ value would give $C_{Y_{\text{total}}}$- Moreover, the first and third terms, which each have a step cost multiplied by the fraction of assemblies made defective in the step of interest, represent auxiliary costs. In equation (6), which shows the sum of all $C_{Y_{\text{step}}}$ values for Fig. 2, the sum of the base costs, $(C_{\text{in}} + C_1 + C_2)/Y_{\text{in}} Y_1 Y_2$, equals the process yielded cost, $C_{Y_{\text{out}}}$, from Fig. 2.

$$C_{Y_{\text{in}}} + C_{Y_1} + C_{Y_2} = \frac{C_{\text{in}} + (1 - Y_{\text{in}})(C_1 + C_2)}{Y_{\text{in}} Y_1 Y_2} + \frac{C_1 + (1 - Y_1)(C_{\text{in}} + C_2)}{Y_{\text{in}} Y_1 Y_2} + \frac{C_2 + (1 - Y_2)(C_{\text{in}} + C_1)}{Y_{\text{in}} Y_1 Y_2} \tag{6}$$

Thus this method gives $C_{Y_{\text{step}}}$ values that can be collected, according to the criteria set previously. In addition, these $C_{Y_{\text{step}}}$ values incorporate upstream and downstream information via the auxiliary costs. In (5), upstream information appears in the $C_{\text{in}}$ term, which represents the cost invested in the assemblies that would be made defective in the first step. In other words, the assemblies made defective in the first step waste a fraction $(1-Y_1)$ of $C_{\text{in}}$. Likewise, the $C_2$ term represents the cost invested in the second step on assemblies made defective in the first step. These assemblies made defective in the first step "absorb" cost from the second step. Furthermore, this approach defines $C_{Y_{\text{step}}}$ values that are independent of step order. In (5), $C_{Y_1}$ would not change if steps 1 and 2 were switched. This is because both base cost and auxiliary cost terms are independent of step order. The base costs only depend on the cost of the base step and the process yield, $Y_{\text{in}} Y_1 Y_2$, which remains the same during step switching. Likewise, both auxiliary cost terms have the same auxiliary yield factor, $(1-Y_1)$, so switching step order will not affect the result. This is intuitive for the following reason. If cost is incurred before step 1, then the fraction $(1-Y_1)$ of assemblies made defective in step 1, force the loss of this incurred cost. Additionally, if cost is incurred after step 1, then these assemblies also absorb a fraction $(1-Y_1)$ of this cost. Either cost is incurred on assemblies that are defective or on assemblies to be made defective and an amount $C_{\text{step}}(1-Y_1)$ of cost is lost due to the defect generation in step 1. For these reasons, auxiliary costs, and thus, step yielded costs, are independent of step order.

Since this method defines step yielded cost values that not only can be collected to get process yielded cost, but that incorporate upstream and downstream information and that are independent of step order for steps between scrapping points, the omission method is the most appropriate of the four methods.
III. Test Operations

When a process flow includes test or inspection steps, some assemblies are removed from the process and are either reworked (Section IV) or scrapped (discarded into the waste stream). Consider the following process (Fig. 3a) that scraps only defective assemblies (100% fault coverage ($f_c = 1$) and zero false positives are assumed). Because the test steps remove all defective assemblies, the yield of assemblies remaining in the process following each test step will be 100% and the fraction of assemblies passing the test step (i.e., the fraction of assemblies not scrapped) will be equal to yield prior to the test step. Furthermore, between each test step, there will be some sequence of assembly steps with some net cost and net yield.

Let $C_{tj}$ be the accumulated cost per assembly following the $j^{th}$ test step and let $Y_{tj}$ be the yield following the $j^{th}$ test step. Then,

$$C_{tj} = [C_1 + C_2 + Y_1(C_3 + C_4) + Y_1Y_2(C_5 + C_6) + ... + \prod_{i=1}^{j} Y_i(C_{2i+1} + C_{2j})]$$

$$Y_{tj} = 1 \text{ (for the 100% fault coverage case assumed above)}$$

The yielded cost following the $j^{th}$ test step, $C_{Ytj}$, is then

$$C_{Ytj} = \frac{C_{tj}}{Y_{tj}}$$

In reality, test steps will miss some defective assemblies (test escapes). The fault coverage fraction, $f_c$, represents the fraction of faults present in an assembly that are successfully detected by the test. For a test step with yields $Y_a$ and $Y_b$ before and after the test respectively,$^1$

$$Y_b = Y_a^{1-f_c}$$

$$P = Y_a^{f_c}$$

The yield following a test step is thus the yield prior to test raised to the exponent 1-$f_c$. Similarly, the fraction of assemblies passing a test step (the pass fraction, $P$) is the previous pass fraction multiplied by the yield prior to test raised to the exponent $f_c$. Using (10) and (11), the expressions in Fig. 3a are rewritten in Fig. 3b. With $f_c = 1$, the expressions in Fig. 3b reduce to those obtained in Fig. 3a.

Additionally, following [19], if the test step itself introduces defects before scrapping, then $Y_{in}$ values in (10) and (11) become $Y_{in}Y_{out}$ because both the originally defective assemblies and the newly defective assemblies will be scrapped. If the test step introduces defects after scrapping, then only (10) is multiplied by $Y_{test}$, while (11) remains the same. Furthermore, for a single test step with faults introduced either before or after scrapping, the process yielded cost is the total cost invested divided by the fraction of good parts passing the test step:

$$C_{Yout} = \frac{C_{out}}{Y_{out}} = \frac{C_{in} + C_{test}}{(Y_{in}Y_{test})^{f_c}(Y_{in}Y_{test})^{(1-f_c)}} \text{ or } \frac{C_{in} + C_{test}}{(Y_{in})^{f_c}(Y_{in})^{(1-f_c)}Y_{test}} = \frac{C_{in} + C_{test}}{Y_{in}Y_{test}}$$

The fraction of good parts passing the test step is expressed in (12) as the fraction of assemblies passing the test multiplied by the fraction of parts that are non-defective. Note, that process yielded cost does not depend on when the test step introduces its defects. If defects are introduced prior to scrapping (the first term in (12)), then more assemblies will be scrapped and there will be less assemblies of higher yield following the test step. Conversely if defects are introduced after scrapping (the second term in (12)), then

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$^1$ Equation (10) is derived in [18], (11) follows through simple derivation.
less assemblies will be scrapped and there will be more assemblies of lower yield following the test step. Either way, the total number of good assemblies following test will be the same. Clearly, however, it is desirable to have defects introduced before scrapping so that fewer total assemblies move through the process ensuring that less money would be spent in subsequent steps. This is handled with the omission method by computing the step yielded cost of the test process.

IV. DISTRIBUTION OF STEP YIELDED COST BY OMISSION METHOD

To see how the omission method distributes $C_{\text{yield}}$, consider the example shown in Fig. 4. The matrix in Fig. 5 shows how base costs and auxiliary costs are distributed among process steps for this process flow. The diagonal elements, in bold, represent base costs while the off-diagonal elements represent auxiliary costs. The sum of the base costs and auxiliary costs in each column are the step yielded costs, and are shown in the “Total” row.

The row headings of Fig. 5 represent where costs are incurred. For example, the value in row 1, column 2 represents the proportion of money spent by step 1 on assemblies that will eventually be made defective in step 2. This term is an auxiliary cost and is the money wasted at step 1 due to the yield of step 2. From the matrix, it can be seen that, aside from step 4, base costs contribute the most towards the step yielded cost. Also, notice how the auxiliary costs in step 4 are relatively high. Because step 4 has the lowest yield, it creates the most defects in assemblies and, thus, will incur the most cost for other steps.

Step 4 is a “bad” step and this is shown by its high step yielded cost. On the other hand, the test step has negative auxiliary costs, which mean that it actually saves money for other steps. The test step is a “good” step, which is shown by its negative step yielded cost.

To make the most effective change in process yielded cost for this example, one should decrease the largest auxiliary cost, $123.01. This can be done either by decreasing the cost of the step 3 or by increasing the yield of step 4. In terms of improving step yields, for linear processes, it turns out to be most efficient to increase the lowest yield in a process, shown by (13).²

$$\frac{d(C_Y)}{dY} = \frac{d(CY^{-1})}{dY} = -CY^{-2}$$

(13)

Equation (13) shows that the rate of change of yielded cost is more negative at lower yields. Thus, yielded cost drops more quickly with increases in yield at lower step yields. It is thus more efficient to improve the yield of step 4 than any other step yield. This is true for linear processes such as this one, but does not always hold for nonlinear processes, [20]. An additional example of how the omission method distributes yielded cost is presented in [20].

V. REWORK

In today’s high-value electronics manufacturing industry, reworking assemblies (repairing defective assemblies and inserting them back into a process) is a reality for many types of applications. When significant money is invested in an assembly, manufacturers cannot afford to dispose of defective assemblies and can justify the expenditure of considerable resources to diagnose and rework assemblies to recover their investment. Of the assemblies scrapped by a test step, a fraction ($f_d$) is repairable while those that are not repaired will be scrapped. Overall, adding diagnosis and rework steps increases the total cost of the system but also improves the final yield. Fig. 6 shows an example process flow [19].

Assuming that defects are introduced prior to test (i.e., the yield of the test step takes effect prior to fault coverage), the yield and number of assemblies that pass the test step on the first rework cycle are given by,

$$Y_{\text{out}} = (Y_{\text{in}} Y_{\text{test}})^{1-f_d}$$

$$N_{\text{out}} = N_{\text{in}} (Y_{\text{in}} Y_{\text{test}})^{f_d}$$

(14)

In most cases, it is most efficient to increase the lowest yield in a process. However, [20] provides a more general way to determine the most efficient method for process improvement.
The assemblies going to the diagnosis/rework step are those that do not pass the test,

\[ N_t = N_{in} (1 - (Y_{in} Y_{test})^{f_t}) \quad (16) \]

The number of assemblies re-entering the test step is the product of the fraction reworked and \( N_t \). Also, the number of assemblies scrapped (those that are not reworked) is given by,

\[ N_{dr} = f_{dr} N_{in} (1 - (Y_{in} Y_{test})^{f_t}) \]
\[ N_{Sdr} = (1 - f_{dr}) N_{in} (1 - (Y_{in} Y_{test})^{f_t}) \quad (17) \]

For purposes of calculating the total cost invested in the \( n^{th} \) test/diagnosis/rework attempt, \( N_{outn} \), \( N_{tn} \), and \( N_{drn} \) are found from (15) through (18). The first rework cycle \((n = 1)\) is defined to begin with the first encounter of the diagnosis/rework step. Assuming that the yield of the assemblies entering the test step will be \( Y_{dr} \) \((Y_{in} \text{ for } n = 0)\), the following general expressions give the number of assemblies leaving each respective step during the \( n^{th} \) rework cycle.

\[ N_{tn} = N_{in} f_{dr} \sum_{n=1}^{n-1} (1 - (Y_{in} Y_{test})^{f_t})(1 - (Y_{dr} Y_{test})^{f_t})^{n-1} \quad (19) \]
\[ N_{drn} = N_{in} f_{dr} \sum_{n=1}^{n-1} (1 - (Y_{in} Y_{test})^{f_t})(1 - (Y_{dr} Y_{test})^{f_t})^{n-1} \quad (20) \]
\[ N_{outn} = N_{in} f_{dr} \sum_{n=1}^{n-1} (1 - (Y_{in} Y_{test})^{f_t})(1 - (Y_{dr} Y_{test})^{f_t})^{n-1} (Y_{dr} Y_{test})^{f_t} \quad (21) \]

The yielded cost of the rework process is the total cost invested by the test and diagnosis/rework steps over \( n \) rework attempts divided by the total number of good assemblies leaving the test step for \( n = 0, 1, \ldots, n \) rework attempts. This effectively represents the cost per good assembly invested by the rework system. The total cost invested for \( n \) rework attempts, \( C_{tn} \), is the total number of assemblies entering each step (or the number of assemblies leaving the previous step), multiplied by each respective step cost,

\[ C_{n t} = C_{test} + \sum_{i=1}^{n} (1 - (Y_{in} Y_{test})^{f_t}) f_{dr} \sum_{n=1}^{n-1} (1 - (Y_{dr} Y_{test})^{f_t})^{n-1} [C_{test} f_{dr} + C_{dr}] \quad (22) \]

Likewise, the total number of good assemblies passing the test step, \( N_g \), is the total number of assemblies passing the test step multiplied by their respective yields,

\[ N_g = (Y_{in} Y_{test} + (Y_{dr} Y_{test})(1 - (Y_{in} Y_{test})^{f_t}) \sum_{n=1}^{n} f_{dr} \sum_{n=1}^{n-1} (1 - (Y_{dr} Y_{test})^{f_t})^{n-1} \quad (23) \]

Therefore, the yielded cost of the entire test/diagnosis/rework process with \( n \) rework attempts is given by,

\[ C_{Y_{tn}} = \frac{C_{tn}}{N_g} \quad (24) \]

Similar results for \( C_{Y_{tn}} \) can be obtained with [19]. Because the model in [19] is more general and treats the diagnosis and rework processes separately, in order to match the results given by (24), one can effectively combine the two steps in the [19] model by either setting the diagnosis cost to zero and the diagnosis fraction to 100% or by setting the rework cost to zero and the rework fraction to 100%.

Consider the example of diagnosis and rework effects on the process shown in Fig. 7. Each plot in Fig. 8 shows the effect of changing one variable and the number of rework attempts on process yielded costs. Plot a) varies the yields of steps 1, 2, 3 and 4 where each series correspondingly decreases each step yield by 0.1. Note that the yielded cost increases exponentially with decreases in step yields. This is shown by
the surface curving upwards from left to right and verifies the claim made in (13) that there exists an exponential relationship. Also, yielded cost decreases with increases in rework attempts, as can be seen in all plots with the surfaces sloping downwards from back to front. Also, note how yielded cost decreases fastest for small numbers of rework attempts. This is true for all the plots and is caused by the fact that rework is most effective in the beginning and becomes less effective with more and more attempts (i.e., a decreasing marginal product of rework attempts). That is, yielded cost converges to a steady-state value rather quickly, even for processes with high costs and low yields. Furthermore, plot b) varies the cost of steps 1, 2, 3 and 4 where each series correspondingly increases each step cost by $50. Notice how the increase in yielded cost is linear. Plot c) varies the fault coverage of the test step, where decreasing fault coverage causes increases in yielded cost. This is because low fault coverage will allow more defective assemblies to pass through test, causing more cost to be incurred for each passing of an assembly. Plot d) varies the rework fraction. Yielded cost increases for decreasing rework fractions because process yield decreases as more and more assemblies are scrapped.

VI. APPLICATION OF YIELDED COST CALCULATIONS TO A SURFACE MOUNT ASSEMBLY PROCESS

In this section an actual process flow was analyzed with the four approaches to calculating yielded cost. Fig. 9 shows all of the yielded cost methods discussed in this paper applied to a series of process steps that represent a simple surface mount assembly operation.

Each approach gives step yielded cost values that differ. For high yield steps, such as place SMT setup, the itemized method values match the step costs. Also, for this example, the itemized method and the cumulative method match closely due to the overall high yields of the process steps. However, the omission method measures step yielded cost completely by taking both base costs and auxiliary costs into account. Therefore, its values are consistently greater than those for the other methods.

VII. SUMMARY

This paper defines and explains yielded cost for simple and complex sequential process flows and develops the concept of yielded cost for test and rework operations. By analyzing existing yielded cost methods, a new model was developed that completely provides information on the effective cost per good assembly for process steps. Two of the existing yielded cost models, the itemized method and iterative method, were deemed to be unreasonable because the CYstep values could not be easily accumulated. Another existing model, the cumulative method was found to be reasonable, but its CYstep values did not incorporate upstream and downstream information and were not independent of step order. Thus the omission method was found to be the most complete approach because it defined CYstep values that incorporated upstream and downstream information and that were independent of step order. The omission method measures the change in process yielded cost when a particular step is removed from a process. Mathematical models were developed for the omission method, its relevance to test and rework situations was explained, and it was further demonstrated on a surface mount assembly process, along with existing yielded cost models. Additionally, the omission method can provide step yielded cost values that allow one to determine how to most efficiently improve a process [20]. To find the best solution to improving the system, an efficiency ratio can be used, where the ratio equals the change in process yielded cost divided by the change in auxiliary yield for a particular step. To best improve a process, one should increase the yield of the step with the highest efficiency ratio [20].

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REFERENCES


Figure Captions

Fig. 1. A simple sequential process flow consisting of m process steps.

Fig. 2. *Cumulative* method: multiple step process.

Fig. 3. A multiple test process with each test step fault coverage equal to 100%. In part b, the generalized expressions are reduced to the results obtained in part a with $f_c = 1$.

Fig. 4. Example test process to demonstrate *omission* method of calculating yielded cost.

Fig. 5. Base costs and auxiliary costs for the process shown in Fig. 4.

Fig. 6. Schematic of a simple rework process.

Fig. 7. Example rework process to demonstrate *omission* method of calculating yielded cost.

Fig. 8. Relationship between total process yielded cost and a) step yields, b) step costs, c) fault coverage, and d) rework fraction for example presented in Fig. 7. Each plot holds the original conditions shown in Fig. 7 and changes one variable. In a), for example, each step yield is decreased by 0.1 (only the values of the first step yield are shown). Similarly, in b) each step cost is increased by $50 (only the values of the first step cost are shown). In c) and d), the fault coverage and rework fraction, respectively, are decreased.

Fig. 9. Demonstration of step yielded costs applied to a surface mount assembly process. The first value in each group is the step cost (a), followed by the step yielded cost calculated with the *itemized* method (b), the *iterative* method (c), the *cumulative* method (d), and the *omission* method (e).
Yielded Cost, \( C_Y = \frac{\text{Cost}}{\text{Yield}} \)

\[
C_Y = \frac{\text{cost per part}}{\text{fraction of good parts}} = \frac{C_{\text{in}} + \sum_{i=1}^{m} C_i}{Y_{\text{in}} \prod_{i=1}^{m} Y_i}
\]

Figure 1

\[
C_{Y_{1\rightarrow 2}} = \frac{C_{\text{in}} + C_1}{Y_1 Y_1}
\]

\[
C_{Y_{\text{out}}} = \frac{C_{\text{out}}}{Y_{\text{out}}} = \frac{C_{\text{in}} + C_1 + C_2}{Y_{\text{in}} Y_1 Y_2}
\]

Figure 2

Total Yield: \( Y_2 \)  
Accumulated Cost: \( C_1 + C_2 + Y_1 C_3 \)

Total Yield: \( Y_3 \)  
Accumulated Cost: \( C_1 + C_2 + Y_1 (C_3 + C_4) + Y_1 Y_2 C_5 \)

Figure 3a
Total Yield: $((Y_1 Y_2)^{1-f_c})Y_3 = Y_3$
Pass Fraction: $Y_1^{f_c} = Y_1$
Accumulated Cost: $C_1 + C_2 + Y_1 Y_3 = C_1 + C_2 + Y_1 Y_3$

Total Yield: $(Y_1 Y_2)^{1-f_c} Y_3 = Y_3$
Pass Fraction: $Y_1^{f_c} ((Y_1 Y_2)^{1-f_c}) Y_3 = Y_2$
Accumulated Cost: $C_1 + C_2 + Y_1 Y_3\left(Y_1 Y_2\right)^{1-f_c} C_5 = C_1 + C_2 + Y_1 Y_3 + Y_1 Y_2 C_5$

Figure 3b

Step 1: $C_1 = 90$
Step 2: $C_2 = 80$
Step 3: $C_3 = 70$
Step 4: $C_4 = 60$

Step that creates defects

<table>
<thead>
<tr>
<th>Step</th>
<th>Cost Incurred</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>$228.70$</td>
<td>7.53</td>
<td>-68.92</td>
<td>68.61</td>
<td>91.48</td>
</tr>
<tr>
<td>Step 2</td>
<td>$3.19$</td>
<td>$203.28$</td>
<td>-61.27</td>
<td>60.99</td>
<td>81.31</td>
</tr>
<tr>
<td>Test</td>
<td>$3.98$</td>
<td>8.36</td>
<td>$254.11$</td>
<td>76.23</td>
<td>101.64</td>
</tr>
<tr>
<td>Step 3</td>
<td>-9.42</td>
<td>-20.80</td>
<td>-108.47</td>
<td>$123.01$</td>
<td>$123.01$</td>
</tr>
<tr>
<td>Step 4</td>
<td>-8.07</td>
<td>-17.83</td>
<td>-92.97</td>
<td>31.63</td>
<td>31.63</td>
</tr>
<tr>
<td>Total</td>
<td>$218.38$</td>
<td>$180.55$</td>
<td>$-77.53$</td>
<td>$360.47$</td>
<td>$429.08$</td>
</tr>
</tbody>
</table>
Figure 6

Test

\[ C_{\text{test}}, Y_{\text{test}}, f_c \]

Diagnosis/Rework

\[ C_{\text{dr}}, Y_{\text{dr}}, f_{\text{dr}} \]

(fraction reworked)

\[ N_{\text{Sdr}} \]

Figure 7

Step 1

\[ C_1 = 90, \quad Y_1 = 0.9 \]

Step 2

\[ C_2 = 80, \quad Y_2 = 0.8 \]

Test

\[ f_c = 0.75, \quad C_{\text{test}} = 100, \quad Y_{\text{test}} = 0.9 \]

Step 3

\[ C_3 = 70, \quad Y_3 = 0.7 \]

Step 4

\[ C_4 = 60, \quad Y_4 = 0.6 \]

Diag/Rwk

\[ f_{\text{dr}} = 0.9 \]

\[ C_{\text{test}} = 100, \quad Y_{\text{test}} = 0.9 \]

Scrap
Figure 8

a) Effect of Step Yields on Yielded Cost

b) Effect of Step Costs on Yielded Cost

c) Effect of Fault Coverage on Yielded Cost

d) Effect of Rework Fraction on Yielded Cost

C_{step1-4} = 90, 80, 70, 60
f_c = 0.75
f_d = 0.9

Y_{step1-4} = 0.9, 0.8, 0.7, 0.6
f_c = 0.75
f_d = 0.9

C_{step1-4} = 90, 80, 70, 60
f_d = 0.9

Y_{step1-4} = 0.9, 0.8, 0.7, 0.6
f_c = 0.75
The first four bars have values of 18.25, the fifth is 21.14